Chiheb Ben Hammouda
KAUST Saudi Arabia
Numerical Methods in Computational Finance

The price of financial derivatives such as options are represented as expectations of random variables, derived from the stochastic models describing the dynamics of the underlying assets. In most of the cases, explicit formulas for the prices are difficult to obtain, i.e. the expectations cannot be computed explicitly. To overcome this issue, many numerical approximations were designed in the finance industry. We can classify these approximation into three general approaches:

1. Monte Carlo based methods: By the law of large numbers, sample averages converge to the expected value of a random variable if the sample size goes to infinity. This property leads to Monte-Carlo simulation and its variants like Quasi Monte-Carlo simulation. In this setting, the main ingredient is to simulate many paths of the underlying asset price. Given that the exact simulation of such paths is usually not possible, approximate simulation methods (e.g. Euler approximations of SDEs) are widely used. MC based method are usually used for high dimensional problems.

2. PDE-based approximation methods: When the dynamics of the underlying asset is given by a Markovian model (e.g. given by an SDE), the option price satisfies a PDE, called the Kolmogorov-backward equation. Therefore, one can compute the option price by solving the PDE numerically using PDE solvers such the finite-difference or finite-element methods. Apart from regularity and (too) exotic path-dependence, the applicability of these family of methods in this context is mainly constrained by the dimension of the underlying (curse of dimensionality).

3. Fourier based methods: The density of the underlying dynamics is usually not known explicitly, and even if it is known, direct quadrature (i.e., numerical approximation) of the integral might be computationally expensive due to smoothness issues. However, in many important cases (e.g., Lévy or affine processes), the Fourier transform of the density (corresponding to the characteristic function of the underlying random variable) is explicitly known, thus allowing to calculate the option price using Fourier methods.

In this mini-course, I will try to cover the above mentioned approaches. More precisely, the content of the course will be a selection of the following:

- Pseudo random numbers (random number generation on the computer)

- Basics of Monte Carlo simulation
- Quasi Monte Carlo
- Monte-Carlo simulation of diffusion models: weak and strong approximations, order of the Euler scheme
- Solving a PDE using finite differences (various finite difference schemes, in particular Crank-Nicolson)
- Option pricing with Fourier methods
- Affine processes and the Heston model.